Exercise 32

Suppose $f(\pi/3) = 4$ and $f'(\pi/3) = -2$, and let $g(x) = f(x)\sin x$ and $h(x) = (\cos x)/f(x)$. Find

(a) $g'(\pi/3)$ (b) $h'(\pi/3)$

Solution

Differentiate $g(x)$ by using the product rule.

$$\begin{aligned} g'(x) &= \frac{d}{dx}[g(x)] \\ &= \frac{d}{dx}[f(x)\sin x] \\ &= f'(x)\sin x + f(x)\left[\frac{d}{dx}(\sin x)\right] \\ &= f'(x)\sin x + f(x)\cos x \end{aligned}$$

Evaluate it at $\pi/3$.

$$g'\left(\frac{\pi}{3}\right) = f'\left(\frac{\pi}{3}\right)\sin\frac{\pi}{3} + f\left(\frac{\pi}{3}\right)\cos\frac{\pi}{3} = (-2)\left(\frac{\sqrt{3}}{2}\right) + (4)\left(\frac{1}{2}\right) = 2 - \sqrt{3}$$

Differentiate $h(x)$ with the quotient rule.

$$\begin{aligned} h'(x) &= \frac{d}{dx}[h(x)] \\ &= \frac{d}{dx}\left[\frac{\cos x}{f(x)}\right] \\ &= \frac{\left[\frac{d}{dx}(\cos x)\right]f(x) - \left[\frac{d}{dx}f(x)\right]\cos x}{[f(x)]^2} \\ &= \frac{(-\sin x)f(x) - f'(x)\cos x}{[f(x)]^2} \\ &= -\frac{f(x)\sin x + f'(x)\cos x}{[f(x)]^2} \end{aligned}$$

Evaluate it at $\pi/3$.

$$h'\left(\frac{\pi}{3}\right) = -\frac{f\left(\frac{\pi}{3}\right)\sin\frac{\pi}{3} + f'\left(\frac{\pi}{3}\right)\cos\frac{\pi}{3}}{[f\left(\frac{\pi}{3}\right)]^2} = -\frac{(4)\left(\frac{\sqrt{3}}{2}\right) + (-2)\left(\frac{1}{2}\right)}{(4)^2} = -\frac{2\sqrt{3} - 1}{16}$$